JOURNAL OF FLUIDS AND STRUCTURES

Journal of Fluids and Structures 19 (2004) 543–555

www.elsevier.nl/locate/jnlabr/yjfls

Hydroelastic analysis of a large floating structure

M. Ohkusu[a,*], Y. Namba[b]

[a] RIAM, Kyushu University, Fukuoka, Japan
[b] National Maritime Research Institute, Tokyo, Japan

Received 1 August 2003; accepted 7 February 2004

Abstract

An analytical approach to predict the bending vibration of a very large floating structure of thin and elongated rectangular plate configuration, floating on water of shallow depth and under the action of a monochromatic head wave, is presented. The horizontal size of the plate is huge compared with the wavelength of the incident waves, yet the wavelength is much larger than the draft. The fluid–plate interaction is solved by considering that the draft of the plate is asymptotically zero and the plate bottom surface is located at the water surface. The boundary condition for the fluid flow at the plate bottom surface is derived from the hydroelastic behavior of the plate and is different from the condition at the real water surface. The solution is constructed by matching the wave in the water surface to the wave on the plate bottom surface, i.e., the transverse vibration of the plate. Solutions valid in three sub-regions of the plate bottom surface and two sub-regions of the water surface are found separately to be asymptotically matched. The plate vibration is obtained in an explicit form and its accuracy is confirmed against the results from more computationally involved approach.

1. Introduction

A design of floating airport proposed in Japan is of a thin mat-like configuration, of horizontal size as large as several kilometers and a relatively small vertical size of less than 10 m. A floating structure of this configuration will be exposed to the sea over almost its entire surface, and consequently wave action will be of considerable magnitude; the bending rigidity will be relatively small and elastic deflection will be crucial for its feasibility. The rigid body motion, on the other hand, will be small and may be disregarded, because the wavelength of sea waves of practical concern is obviously too short compared with the huge size of the structure to induce rigid body motions.

Accurate prediction of the interaction of such a thin but huge floating structure with water waves is rather a new problem. Approaches used for analysis of wave-ship or wave–structure interaction [e.g., Faltinsen (1990)] will be applicable but may not always be the best for straightforward prediction of the elastic response of such a peculiar configuration. Several efforts (Newman et al., 1996; Kashiwagi, 1998), however, have been published, concerning numerical computation of the elastic response by implementing the boundary element method.

An alternative model, accounting for the fact that the body is very thin and the wave length is very short compared with its horizontal dimension, leads us to more analytical approaches (Evans and Davies, 1968; Meylan and Squire, 1994; Ohkusu and Namba, 1996, 1998; Hermans, 1997; Namba and Ohkusu, 1999). The analytical approach is more lucid than the numerical one, and yields an explicit mathematical expression for the elastic response with almost no burden of numerical work, though its application will be limited to a simple body geometry, i.e., a rectangle. A numerical procedure based on the boundary element method will of course be required when we are concerned with

*Corresponding author.
E-mail address: ohkusu@riam.kyushu-u.ac.jp (M. Ohkusu).

0889-9746/$ - see front matter
doi:10.1016/j.jfluidstructs.2004.02.002

actual design of this type of floating structure which is inevitably of complicated configuration, and not an exact rectangle. Nevertheless, the analytical approach will be a better tool with which to understand vibration in waves of such a unusual structure. Moreover, the results will be useful as a validation of more computationally involved boundary element methods.

This paper is concerned with an analytical approach to predict vibration of a very large floating structure (VLFS) of thin and rectangular plate configuration floating on water of shallow depth. We seek a solution as the draft of the structure approaches zero. Employment of the linear shallow water theory in addition to a small draft approximation facilitates simpler formulation of our problem. A similar formulation was proposed by Stoker (1958) for two-dimensional analysis of a floating elastic body. The shallow water assumption will be rather realistic, because floating airports are intended to be installed in coastal areas. A similar type of analysis is possible for deep water, though more computational work would be required (Ohkusu and Namba, 1996, 1998).

We assume the waves are incident head on to the structure (for oblique incidence Namba and Ohkusu (1999) gave a result for infinitely long plates and Takagi (2001) considered a plate of finite length). If the width of the structure is of the same order as the incident wave length, the analysis is less complicated with the assumption of slender body diffraction (Ohkusu and Namba, 1998). The width of an actual airport, however, will be very large compared with the wavelength of incident waves. In this paper, we study the case in which both the width and the length of the structure are very large, yet the former is much smaller than the latter.

2. Formulation of the problem

Suppose an elongated rectangular plate is floating at a uniform draft d in equilibrium, as shown in Fig. 1. The length L and width B are as large as kilometers while the draft d is a few meters. The water depth h is assumed uniform. The thickness of the plate, which does not appear explicitly in our analysis, will be of the order $\mathcal{O}(d)$. Let (x, y, z) be rectangular coordinates with the x–y plane in the calm water surface and with the z-axis vertically upwards. Let ρ and g denote density of the water and gravitational acceleration.

We study transverse vibration of the plate under the action of a monochromatic incident wave of amplitude a,

$$\zeta_0 e^{i\omega t} = a e^{-i(kx-\omega t)}. \tag{1}$$

Here ω is the wave frequency and k the wavenumber.

We consider the flow and oscillation of the plate to be time-periodic: the velocity potential is of the form $\Phi(x, y, z)e^{i\omega t}$. Assuming the oscillation is small, Φ should satisfy the linearized body boundary condition on the plate surface in equilibrium, the water bottom condition at $z = -h$ and the linearized free surface condition on the mean water surface $z = 0$.

Let $\zeta(x, y)e^{i\omega t}$ denote the plate transverse oscillation which is to be determined from the fluid–plate interaction analysis. The body boundary condition on the bottom surface of the plate is given by

$$\frac{\partial \Phi}{\partial z} = i\omega\zeta \quad \text{on } z = 0. \tag{2}$$

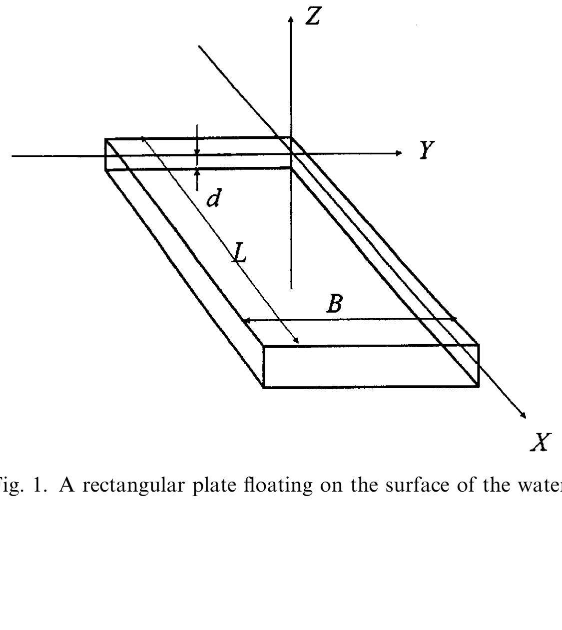

Fig. 1. A rectangular plate floating on the surface of the water.

In fact this condition is to be imposed at the correct bottom surface $z = -d$, but since we assume d is very small compared with any other length scale, it may be transferred to $z = 0$. Another body boundary condition on the side surface $-d < z < 0$ at the four edges of the plate will be required to be satisfied. Its effect, however, is limited to within a very small distance of the order $\mathcal{O}(d)$ from the edges, and Φ determined under only condition (2) will be a good estimate for the potential throughout most of the fluid region. Obviously, one need not account for the body boundary condition at the side surface when the transverse oscillation of the plate is concerned (this will not be true when one intends to predict the hydrodynamic force acting in the horizontal direction).

The dimensions of the plate and the wave length of practical concern will justify the assumptions:

$$kh \ll 1 \ll kB, kL. \tag{3}$$

One of the most probable applications of this type of platform will be a floating airport supposed to be located in a coastal area, where the assumption of shallow water will be plausible.

We employ the linear shallow water theory [e.g., Stoker (1958)]. Let a function $\phi(x, y)$ be $\Phi(x, y, 0)$. Then ϕ satisfies a linear wave equation in the water region, the top of which is not covered by the plate:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi + k^2\phi = 0, \tag{4}$$

where $k = \omega/\sqrt{gh}$. The body boundary condition (2) is rewritten as

$$\mathrm{i}\omega\zeta = \frac{\partial \Phi}{\partial z}_{z=0} = -h\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi. \tag{5}$$

When we interpret $\zeta(x, y)\mathrm{e}^{\mathrm{i}\omega t}$ as the wave elevation in the water surface not covered by the plate, Eq. (5) is valid in giving the wave elevation in terms of ϕ.

The equation of small transverse vibration of the plate is

$$D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \zeta\mathrm{e}^{\mathrm{i}\omega t} - m\omega^2\zeta\mathrm{e}^{\mathrm{i}\omega t} = \rho g\zeta\mathrm{e}^{\mathrm{i}\omega t} - \rho\mathrm{i}\omega\phi\mathrm{e}^{\mathrm{i}\omega t}. \tag{6}$$

Here D denotes the bending rigidity of the plate per unit length. The second term accounts for the inertia effect of the plate, where m is the mass of the plate per unit area. The weight must initially balance the buoyancy of the plate and m is proportional to d. The second term is thus of order $\mathcal{O}(kd \cdot kh)$ which is small enough to be ignored compared with other terms, although its inclusion does not cause any analytical difficulty. The third term represents the variation of the buoyancy due to the local displacement of the plate surface from equilibrium. The fourth term is the dynamical fluid pressure acting on the plate bottom. The first term may need a little more discussion. Clearly only $D\nabla^4 = \mathcal{O}(1)$ (∇^2 denotes the Laplacian in the x–y plane) produces a consistent and practically meaningful solution to the plate transverse vibration equation. Since the spacial variation of the plate deflection is of the same order $\mathcal{O}(k)$ as that of the water wave, $D\nabla^4 = \mathcal{O}(1)$ requires the small bending rigidity D to be of order $\mathcal{O}((kL)^{-4})$, when normalized by L. This will be realized if the plate has large length and small thickness.

Multiplying both sides of Eq. (6) by $\mathrm{i}\omega$ and eliminating ζ by introducing Eq. (5), we have the fluid–plate coupling condition underneath the plate:

$$(D\nabla^6 + \rho g\nabla^2 + \rho g k^2)\phi = 0. \tag{7}$$

Other conditions ϕ should satisfy are the radiation condition requiring that the wave motion except for the incident wave should progress outward as $\sqrt{x^2 + y^2} \to \infty$, and the free–free plate condition of zero shear force and zero bending moment at the plate edges.

In summary our problem is: (i) to determine $\phi(x, y)$ satisfying Eq. (4) in the water region not covered by the plate, Eq. (7) in the water region underneath the plate, the radiation condition and the free–free plate condition; (ii) to compute the transverse vibration of the plate ζ using Eq. (5).

3. Method of solution

We first study a simplified mathematical problem in which Eq. (7) should be satisfied over a region $\Omega\{(x, y)|\ 0 \leqslant x < +\infty, -\infty < y \leqslant 0\}$ of the x–y plane as shown in Fig. 2; the conditions at two plate edges $x = L$ and $y = -B$ are replaced by the requirement that ϕ be bounded at $x = +\infty$ and $y = -\infty$ as long as it is on Ω. It will be seen later that the result is readily extended to obtain the transverse vibration of an elongated rectangular plate.

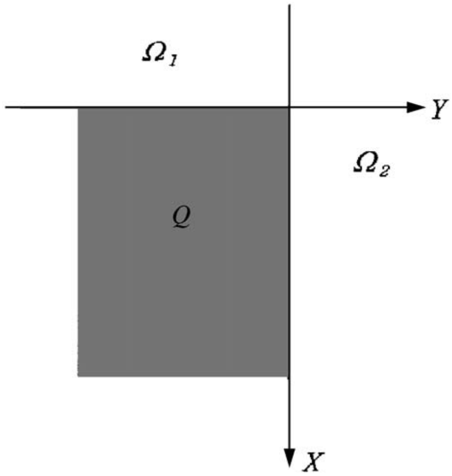

Fig. 2. A thin plate covering a quadrant of the water surface.

In the regions $\Omega_1\{(x,y)| -\infty < x \leqslant 0, -\infty < y \leqslant 0\}$ and $\Omega_2\{(x,y)| -\infty < x < +\infty, 0 \leqslant y\}$ not covered by the plate, condition (4) is to be satisfied.

The lowest order term in an asymptotic expansion of the solution with respect to the reciprocal of large wavenumber k will be

$$\phi = \begin{cases} A_1 e^{-ik_A x} + A_2 e^{\lambda_2 x} + A_3 e^{\lambda_3 x} & \text{in } \Omega, \\ e^{-ikx} + R e^{ikx} & \text{in } \Omega_1, \\ e^{-ikx} & \text{in } \Omega_2. \end{cases} \tag{8}$$

The first and second lines of Eq. (8) for the regions Ω and Ω_1 would represent the solution if the plate had no edge on $y = 0$. k_A is a real positive constant and $\lambda_{2,3}$ are two conjugate complex numbers with negative real part; A_j $(j = 1, 2, 3)$ and R are complex constants (see Appendix for the details). To simplify Eq. (8), a in Eq. (1) has been chosen as $-i\omega/h$. Obviously solution (8) is discontinuous on $y = 0$ corresponding to the division of the plane into the region Ω_1 in which the incident wave and the reflection in front of the plate coexist, the region Ω_2 in which the incident wave is transmitted past the corner of the plate, and the plate region Ω. Naturally, Eq. (8) does not satisfy the edge condition of zero bending moment and zero shear force of the plate.

Solution (8) may be interpreted as an outer solution. We seek an inner solution valid close to the plate edge at $y = 0$ and to be matched with the outer solution. For the sake of simplicity, we ignore the effect of the corner of the plate at $x = 0, y = 0$. This means our analysis is valid for the plate vibration at a location far away (in the scale of wavelength) from the corner.

It is reasonable to attempt the following form of the inner solution:

$$\phi = \psi_0(x,y)e^{-ikx} + \psi_1(x,y)e^{-ik_A x} + \psi_2(x,y)e^{\lambda_2 x} + \psi_3(x,y)e^{\lambda_3 x}, \tag{9}$$

assuming the slow variation of ψ_j $(j = 0, 1, 2, 3)$ in the x direction compared with the variation in the y direction. Hereafter we denote $\lambda_0 = -ik$ and $\lambda_1 = -ik_A$ when it is convenient.

3.1. Solution in the region $y = \mathcal{O}(k^{-1})$ of Ω

In this region the variation of ψ_j in the y direction will be as large as

$$\frac{\partial \psi_j}{\partial y} = \mathcal{O}(k\psi_j). \tag{10}$$

The conditions of zero shear force and zero bending moment at $y = 0$, $x > 0$ are written for ϕ:

$$\frac{\partial}{\partial y}\left[\nabla^4 + (1-\nu)\frac{\partial^2}{\partial x^2}\nabla^2\right]\phi = 0, \quad \left[\nabla^4 - (1-\nu)\frac{\partial^2}{\partial x^2}\nabla^2\right]\phi = 0, \tag{11}$$

where v is Poisson's ratio of the plate. It will be readily seen from the magnitudes of the derivatives of ϕ in the x direction that the plate edge conditions would not be satisfied unless the variation of ϕ in the y direction satisfies condition (10).

Substituting Eq. (9) into Eq. (7) and retaining the lowest order terms, in view of Eq. (10), we have

$$D\frac{\partial^6\psi_j}{\partial y^6} + 3D\lambda_j^2\frac{\partial^4\psi_j}{\partial y^4} + (\rho g + 3D\lambda_j^4)\frac{\partial^2\psi_j}{\partial y^2} = 0 \quad (j = 1, 2, 3), \tag{12}$$

$$D\frac{\partial^6\psi_0}{\partial y^6} - 3Dk^2\frac{\partial^4\psi_0}{\partial y^4} + (\rho g + 3Dk^4)\frac{\partial^2\psi_0}{\partial y^2} - Dk^6 = 0. \tag{13}$$

A solution of Eq. (12) is written as

$$\psi_j = \sum_{m=1}^{4} E_{jm}(x)\mathrm{e}^{\sigma_{jm}y} + a_j(x)y + b_j(x) \quad (j = 1, 2, 3), \tag{14}$$

where E_{jm} and a_j, b_j are independent of y; σ_{jm} are the four roots of the equation

$$D\sigma_j^4 + 3D\lambda_j^2\sigma_j^2 + (\rho g + 3D\lambda_j^4) = 0. \tag{15}$$

The terms of the series in Eq. (14) that grow exponentially as y approaches $-\infty$ are to be excluded, because it should match with a solution at $y = -\infty$, as shown later; two roots of Eq. (15) are of negative real part ($\mathscr{Re}(\sigma_{j3})<0$ and $\mathscr{Re}(\sigma_{j4})<0$) and therefore E_{j3} and E_{j4} must be zero. σ_{j1} and σ_{j2} for the nonzero terms are given by

$$\sigma_{j1}, \sigma_{j2} = \frac{1}{\sqrt{2}}\left(-3\lambda_j^2 \pm \mathrm{i}\sqrt{4\rho g/D + 3\lambda_j^4}\right)^{\frac{1}{2}}, \quad |\arg(\sigma_{j1})|, |\arg(\sigma_{j2})| < \pi/2. \tag{16}$$

A solution of Eq. (13) will be written in the form

$$\psi_0 = \sum_{m=1}^{6} E_{0m}\mathrm{e}^{\sigma_{0m}y}, \tag{17}$$

where σ_{0m} are the six roots of the equation

$$D(\sigma_0^2 - k^2)^3 + \rho g(\sigma_0^2 - k^2) + \rho g k^2 = 0. \tag{18}$$

Obviously three of them, σ_{01}, σ_{02} and σ_{03}, have positive real part and the others have negative real part; E_{04}, E_{05} and E_{05} are zero for the same reason that E_{j3} and E_{j4} are zero. For the first three terms

$$\sigma_{01} = \sqrt{k^2 - k_\Lambda^2}, \quad \sigma_{02} = \sqrt{k^2 + \lambda^2}, \quad \sigma_{03} = \sigma_{02}^* \tag{19}$$

(see Appendix for the definition of the complex number λ), where the asterisk stands for the complex conjugate.

Conditions (11) at the plate edge ($y = 0$) ensuring free shear force and free bending moment are written for each ψ_j $(j = 0, 1, 2, 3)$ as

$$\left[\left(\lambda_j^2 + \frac{\partial^2}{\partial y^2}\right)^2 + \lambda_j^2(1 - v)\left(\lambda_j^2 + \frac{\partial^2}{\partial y^2}\right)\right]\frac{\partial\psi_j}{\partial y} = 0, \tag{20}$$

$$\left[\left(\lambda_j^2 + \frac{\partial^2}{\partial y^2}\right)^2 - \lambda_j^2(1 - v)\left(\lambda_j^2 + \frac{\partial^2}{\partial y^2}\right)\right]\psi_j = 0. \tag{21}$$

It is clear now from these equations that assumption (10) is legitimate.

Letting

$$P_{jm} = [(\sigma_{jm}^2 + \lambda_j^2)^2 + (1 - v)\lambda_j^2(\sigma_{jm}^2 + \lambda_j^2)]\sigma_{jm}, \tag{22}$$

$$Q_{jm} = [(\sigma_{jm}^2 + \lambda_j^2)^2 - (1 - v)\lambda_j^2(\sigma_{jm}^2 + \lambda_j^2)], \tag{23}$$

we rewrite the plate edge conditions (20) and (21) to obtain:

$$P_{01}E_{01}(x) + P_{02}E_{02}(x) + P_{03}E_{03}(x) = 0, \tag{24}$$

$$Q_{01}E_{01}(x) + Q_{02}E_{02}(x) + Q_{03}E_{03}(x) = 0, \tag{25}$$

$$P_{j1}E_{j1}(x)+P_{j2}E_{j2}(x)+\lambda_j^4(2-\nu)a_j(x)=0, \tag{26}$$

$$Q_{j1}E_{j1}(x)+Q_{j2}E_{j2}(x)+\lambda_j^4\nu b_j(x)=0, \tag{27}$$

where Eqs. (26) and (27) are for $j=1,2$ and 3.

Eqs. (24)–(27) are not sufficient to determine all the unknowns E_{jm}, a_j and b_j; other conditions will be provided by matching with the solutions in the other regions.

3.2. Solution in the region $y=\mathcal{O}(k^{-1/2})$ in Ω

It is obvious that the inner solution (14) when letting y approach $-\infty$, does not match with the corresponding term in the first line of the outer solution (8) in Ω. We need a solution bridging both solutions; it should be constructed in the region $y=\mathcal{O}(k^{-1/2})$ in Ω. On the other hand ψ_0 given by Eq. (17) becomes zero as y goes to $-\infty$; it is consistent with the outer solution (8) which has no e^{-ikx} component in Ω.

In the region $y\sim-\mathcal{O}(k^{-1/2})$ the variation of ψ_j ($j=1,2,3$) will be a little larger in the y direction than in the x direction:

$$\frac{\partial\psi_j}{\partial y}=\mathcal{O}(k^{1/2}\psi_j),\quad \frac{\partial\psi_j}{\partial x}=\mathcal{O}(\psi_j). \tag{28}$$

The lowest order terms of Eq. (7), when Eq. (9) has been substituted and Eq. (28) assumed, yields

$$(\rho g+3D\lambda_j^4)\left(\frac{\partial^2\psi_j}{\partial y^2}+2\lambda_j\frac{\partial\psi_j}{\partial x}\right)=0. \tag{29}$$

In view of the anticipated matching with the outer solution (8) in the region Ω, a solution of Eq. (29) can be written in the form (Mei and Tuck, 1980)

$$\psi_j=A_j+\frac{1}{i\varepsilon_j\sqrt{2\pi\lambda_j}}\int_0^x d\xi\,\frac{V_j(\xi)}{\sqrt{x-\xi}}e^{\lambda_j y^2/2(x-\xi)}, \tag{30}$$

in which unknown functions $V_j(x)$ are to be determined later by matching with solution (14). Here $\varepsilon_{1,3}=-1$ and $\varepsilon_2=1$. In deriving this solution we have assumed $\psi_j\to A_j$ as x approaches zero.

3.3. Solution in the water region Ω_2

The solution ψ_j ($j=1,2,3$) in the water region Ω_2 is understood to correspond to the wave generated by the plate vibration, while the solution ψ_0 represents the diffraction at the plate edge $y=0$ of the outer wave given by the third line of Eq. (8). The ψ_j ($j=1,2,3$) are easily found. Substituting Eq. (9) into the wave equation (4) and considering the slower variation of ψ_j in the x direction than in the y direction, we obtain

$$\frac{\partial^2\psi_j}{\partial y^2}+(k^2+\lambda_j^2)\psi_j=0 \tag{31}$$

for $j=1,2$ and 3. We may choose the following solution of Eq. (31) that is valid in the water region Ω_2 near the plate edge at $y=0$:

$$\psi_j=T_j(x)\exp\left[i(-1)^j\sqrt{k^2+\lambda_j^2}\,y\right]\quad (j=1,2,3), \tag{32}$$

where T_j are to be determined by matching with the solution in the other region. For ψ_0, on the other hand, we should assume a little smaller variation in the y direction to obtain a nontrivial solution. Then, we have

$$-2ik\frac{\partial\psi_0}{\partial x}+\frac{\partial^2\psi_0}{\partial y^2}=0. \tag{33}$$

After some algebra we obtain the solution of Eq. (33) as

$$\psi_0=1-\frac{1-i}{2\sqrt{\pi k}}\int_0^x d\xi\,\frac{V_0(\xi)}{\sqrt{x-\xi}}e^{-iky^2/2(x-\xi)}. \tag{34}$$

We can match Eqs. (32) and (34) with the outer solution (8) in the region Ω_2. ψ_j given by Eq. (32) becomes zero as y approaches $+\infty$ since the real part of $i(-1)^j\sqrt{k^2+\lambda_j^2}$ is negative for $j=2$ and 3; and the wave corresponding to $j=1$, which is generated by the plate vibrating in the progressive mode $\psi_1 e^{-ik_\Lambda x}$, does not exist where $y > x\tan(\sqrt{k^2-k_\Lambda^2}/k)$ (as long as the plate corner effect is ignored). ψ_0 given by Eq. (34), is associated with the interaction of the plate edge and the incident waves. It is constructed such that it approaches unity as y goes to infinity, and is matched with the third line of the outer solution (8).

3.4. Completion of matching

In order to determine all the unknowns appearing in Eqs. (14), (17), (30), (32) and (34), which are valid in their respective regions, we have to complete the matching process. The matching and other conditions are summarized as follows:

(a) the inner solution (30) and solution (17) are matched with the first line of the outer solution (8) in the plate region Ω;
(b) the inner solution (30) is matched with the inner–inner solutions (14) in the region Ω;
(c) solutions (14) and (17) in the region Ω are continuous at $y=0$ with solutions (32) and (34) in the region Ω_2;
(d) zero bending moment and zero shear force at the plate edge of $y=0$ are satisfied with solutions (14) and (17);
(e) solutions (32) and (34) are matched with the third line of the outer solution (8) in the region Ω_2.

Requirements (a) and (e) are met because solutions (17), (30), (32) and (34) were thus constructed. Condition (d) was already given by Eqs. (24)–(27). The remaining conditions to be satisfied are (b) and (c).

3.4.1. The case $j \neq 0$

As $y \to -\infty$, Eq. (14) approaches

$$\psi_j = a_j(x)y + b_j(x). \tag{35}$$

When $y \to 0$, then Eq. (30) will be

$$\psi_j = A_j + \frac{1}{i\varepsilon_j\sqrt{2\pi\lambda_j}}\int_0^x d\xi\,\frac{V_j(\xi)}{\sqrt{x-\xi}} - yV_j(x). \tag{36}$$

Eqs. (35) and (36) should match by the requirement of (b). We have

$$a_j(x) + V_j(x) = 0, \tag{37}$$

$$b_j(x) - \frac{1}{i\varepsilon_j\sqrt{2\pi\lambda_j}}\int_0^x d\xi\,\frac{V_j(\xi)}{\sqrt{x-\xi}} = A_j. \tag{38}$$

Condition (c) will require the continuity of Eqs. (14) and (32) and of their normal fluxes at $y=0$. This yields

$$E_{j1}(x) + E_{j2}(x) + b_j(x) - T_j(x) = 0, \tag{39}$$

$$\sigma_{j1}E_{j1}(x) + \sigma_{j2}E_{j2}(x) + a_j(x) - (-1)^j i\sqrt{k^2+\lambda_j^2}\cdot T_j(x) = 0. \tag{40}$$

Eqs. (26), (27), (37), (38), (39) and (40) lead to relations between the unknowns $E_{j1}(x)$, $E_{j2}(x)$, $a_j(x)$, $b_j(x)$, $T_j(x)$ and $V_j(x)$ as follows:

$$E_{j1} = -(M_j^{-1})_{15}V_j(x), \quad E_{j2} = -(M_j^{-1})_{25}V_j(x), \tag{41}$$

$$a_j = -(M_j^{-1})_{35}V_j(x), \quad b_j = -(M_j^{-1})_{45}V_j(x), \quad T_j = -(M_j^{-1})_{55}V_j(x), \tag{42}$$

where M_j^{-1} is the inverse of matrix M_j:

$$M_j = \begin{pmatrix} P_{j1} & P_{j2} & \lambda_j^4(2-\nu) & 0 & 0 \\ Q_{j1} & Q_{j2} & 0 & \lambda_j^4\nu & 0 \\ 1 & 1 & 0 & 1 & -1 \\ \sigma_{j1} & \sigma_{j2} & 1 & 0 & (-1)^{j+1}i\sqrt{k^2+\lambda_j^2} \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \tag{43}$$

Substitution of b_j given by Eqs. (42) into Eq. (38) yields an Abel integral equation of the second kind for $V_j(x)$:

$$Z_j V_j(x) - \frac{1}{\mathrm{i}\varepsilon_j\sqrt{2\pi\lambda_j}} \int_0^x \mathrm{d}\xi \frac{V_j(\xi)}{\sqrt{x-\xi}} = A_j, \tag{44}$$

where $Z_j = -(M_j^{-1})_{45}$. The exact solution of Eq. (44) is given by

$$V_j(x) = \frac{A_j}{Z_j} \exp\left[-\frac{x}{2\lambda_j Z_j^2}\right] \mathrm{erfc}\left(-\frac{\sqrt{x/2}}{\mathrm{i}\varepsilon_j\lambda_j}\right), \tag{45}$$

where erfc is the complementary error function. Once $V_j(x)$ is known, all other unknowns are readily determined from Eqs. (41) and (42).

3.4.2. The case $j = 0$

The continuity of ϕ and the normal flux is required also for Eqs. (17) and (34). In the limit of $y \to 0$, Eq. (34) reduces to

$$\psi_0 = \left(1 - \frac{1-\mathrm{i}}{2\sqrt{\pi k}} \int_0^x \mathrm{d}\xi \frac{V_0(\xi)}{\sqrt{x-\xi}}\right) + yV_0(x). \tag{46}$$

Then the conditions of continuity of Eqs. (17) and (34) are written as

$$E_{01}(x) + E_{02}(x) + E_{03}(x) = 1 - \frac{1-\mathrm{i}}{2\sqrt{\pi k}} \int_0^x \mathrm{d}\xi \frac{V_0(\xi)}{\sqrt{x-\xi}}, \tag{47}$$

$$\sigma_{01}E_{01}(x) + \sigma_{02}E_{02}(x) + \sigma_{03}E_{03}(x) = V_0(x). \tag{48}$$

Eqs. (24), (25), (47) and (48) determine the unknowns $E_{01}(x), E_{02}(x), E_{03}(x)$ and $V_0(x)$. They are

$$V_0(x) = \frac{1}{T} \mathrm{e}^{-\mathrm{i}x/2kZ_0^2} \mathrm{erfc}\left(\frac{1-\mathrm{i}}{2\sqrt{k}Z_0}\sqrt{x}\right), \tag{49}$$

where $Z_0 = (M_0^{-1})_{13} + (M_0^{-1})_{23} + (M_0^{-1})_{33}$ and M_0^{-1} is the inverse of matrix M_0

$$M_0 = \begin{pmatrix} P_{01} & P_{02} & P_{03} \\ Q_{01} & Q_{02} & Q_{03} \\ \sigma_{01} & \sigma_{02} & \sigma_{03} \end{pmatrix} \tag{50}$$

and

$$E_{0m} = -(M_0^{-1})_{m3} V_0(x). \tag{51}$$

4. Deflection of the plate

An expression for ψ_j $(j = 1, 2, 3)$ valid in the whole plate region Ω is constructed as a composite expression of Eqs. (8), (14) and (30). ψ_0 is given by a composite of Eqs. (8) and (17). ϕ of Eq. (9), computed by the resulting composite expressions for ψ_j $(j = 0, 1, 2, 3)$, is substituted into Eq. (5) to yield the plate deflection $\zeta(x, y)\mathrm{e}^{\mathrm{i}\omega t}$:

$$\begin{aligned}\zeta(x,y) = &+\frac{\mathrm{i}h}{\omega}\mathrm{e}^{-\mathrm{i}kx}\sum_{m=1}^{3}(\sigma_{0m}^2 - k^2)E_{0m}(x)\mathrm{e}^{\sigma_{0m}y} + \frac{\mathrm{i}h}{\omega}\mathrm{e}^{-\mathrm{i}k_\Lambda x}\left[\sum_{m=1}^{2}(\sigma_{1m}^2 - k_\Lambda^2)E_{1m}(x)\,\mathrm{e}^{\sigma_{1m}y}\right. \\ &\left.-k_\Lambda^2\left(A_1 - \frac{1-\mathrm{i}}{2\sqrt{\pi k_\Lambda}}\int_0^x \mathrm{d}\xi\frac{V_1(\xi)}{\sqrt{x-\xi}}\mathrm{e}^{-\mathrm{i}k_\Lambda y^2/2(x-\xi)}\right)\right] \\ &+\frac{\mathrm{i}h}{\omega}\sum_{j=2}^{3}\mathrm{e}^{\lambda_j x}\left[\sum_{m=1}^{2}(\sigma_{jm}^2 + \lambda_j^2)E_{jm}(x)\mathrm{e}^{\sigma_{2m}y} + \lambda_j^2\left(A_j + \frac{1}{2\sqrt{\pi\lambda_j}}\int_0^x \mathrm{d}\xi\frac{V_j(\xi)}{\sqrt{x-\xi}}\mathrm{e}^{\lambda_j y^2/2(x-\xi)}\right)\right].\end{aligned} \tag{52}$$

As $\lambda_{2,3}$ has a negative real part, the third line of Eq. (52) attenuates quickly away from the front edge of the plate; the plate deflection computed from the first and second lines prevails in most parts of Ω.

Eq. (52) gives the deflection of the plate in the semi-infinite region Ω. Vibration of an elongated plate of width B, however, is readily computed with $\zeta(x,y)$ given by Eq. (52). If Ω has another edge at $y=-B$, its deflection may be computed from

$$\zeta(x,y)+\zeta(x,-y-B) \tag{53}$$

as the deflection is symmetrical about the center line $y=-B/2$ of the plate.

The wave elevation in the water region Ω_2 is computed in a similar manner. The wave function ϕ is derived from Eqs. (32) and (34). Then the wave elevation is given by

$$\begin{aligned}\zeta(x,y)=&-\frac{\mathrm{i}h}{\omega}k^2T_1(x)\mathrm{e}^{-\mathrm{i}(k_\Lambda x+\sqrt{k^2-k_\Lambda^2}y)}-\frac{\mathrm{i}h}{\omega}k^2\mathrm{e}^{-\mathrm{i}kx}\left[1-\frac{1-\mathrm{i}}{2\sqrt{\pi k}}\int_0^x\mathrm{d}\xi\,\frac{V_0(\xi)}{\sqrt{x-\xi}}\mathrm{e}^{-\mathrm{i}ky^2/2(x-\xi)}\right]\\&-\frac{\mathrm{i}h}{\omega}k^2\sum_{j=2}^{3}T_j(x)\mathrm{e}^{\mathrm{i}(\lambda_jx+(-1)^j\sqrt{k^2+\lambda_j^2}y)}.\end{aligned} \tag{54}$$

The first line will be dominant except near the front edge of the plate.

One can compute the plate deflection and the wave elevation around the plate from Eqs. (52) and (54) without much numerical work. Numerical work is only required to evaluate the integrals containing $V_j(x)$ (derived by Eqs. (45) and (49)).

5. Results and discussion

In our analysis the principal scale of the plate dimension is the width B; the length L does not appear. Results shown here are, however, normalized and illustrated with a virtual length $L=5B$. This is for convenience in comparing with other results. They are computed for a plate of normalized bending rigidity $D/\rho gL^4=3.2\times10^{-8}$ and Poisson's ratio $\nu=0.3$, which are taken from those values often used (Kashiwagi, 1998) for study of a prototype floating airport.

Fig. 3 is the pattern of the deflection of a thin plate, for a water depth to plate length ratio $h/L=0.01$ and a normalized frequency of the incident waves $KL\equiv\omega^2/gL=100\pi$, which corresponds to $kh=1.77$. The deflection is normalized by the amplitude of the incident waves. It is seen in this figure that the edge effect is a 'parabolic' feature, as expected from Eq. (29) or (30). The front edge effect is localized in the vicinity of the front edge. Since our formulation assumes that no rear edge of the plate exists, no reflection from the rear edge is seen in the plate deflection.

At a lower frequency of $KL=40\pi$ and at the same water depth ($kh=1.1$) a snapshot of the plate deflection computed by the present approach (left) is compared in Fig. 4 with a result by a more complicated numerical approach (right) based on the boundary element method (BE) (Kashiwagi, 1998). The deflection predicted by the present approach appears to be in good agreement with the one by the BE result, though the former predicts a slightly too strong side-edge effect. The present approach ignored the corner effect of the plate, while the BE method accounted for it correctly. Agreement of both results reveals that the corner effect might not be significant.

It must be noted that the number of unknowns to be solved of the BE approach based on the crude zeroth order panel method would be $\mathcal{O}(10^4)$ to obtain this result. In Kashiwagi (1998), however, a bicubic B spline function is utilized to represent the hydrodynamic pressure on the plate, and the number of unknowns is reduced drastically; yet one has to solve for several hundreds of unknowns, including the mode functions of the deflection up to the several hundredth. The present approach, on the other hand, does not need much numerical computation except in evaluating a single integral of nonsingular behavior.

Fig. 5 is a similar comparison at shallower water depth ($kh=0.5$). One sees no specific difference from the type of behavior observed in Fig. 4.

It will be seen in all the figures that a component of the wave number k_Λ is dominant in the plate deflection except at the edge region near $y=0$ or $y=-B$. Numerically the k component is about 10% of the k_Λ component at $k_\Lambda x=2\pi$ and 8 percent at $k_\Lambda x=10\pi$ near the edge, for the cases of Figs. 3 and 4. Other components are all less than 1%. For the shallower water depth shown in Fig. 5 the k component is about 1% of the k_Λ component, even near the edge.

In order to see more clearly the difference between the deflection predicted by the BE method and the present method, the amplitude of the plate deflection at the center-line $y=-B/2$ (left) and at the side edge $y=0$ (right) are compared in Figs. 6 and 7. Fig. 6 corresponds to Fig. 4 and Fig. 7 to Fig. 5. A small fluctuation seen in the result from the BE method is the effect of the deflection wave reflected at the rear end of the plate; the wave progressing in the positive x direction (the main part of the deflection) interacts with the reflected wave of smaller amplitude going in the negative x direction, to produce small standing-wave-like deflection superposed on the main part. Since the present method, assuming the plate to be half-infinitely long in the x direction, does not account for the rear end effect, this

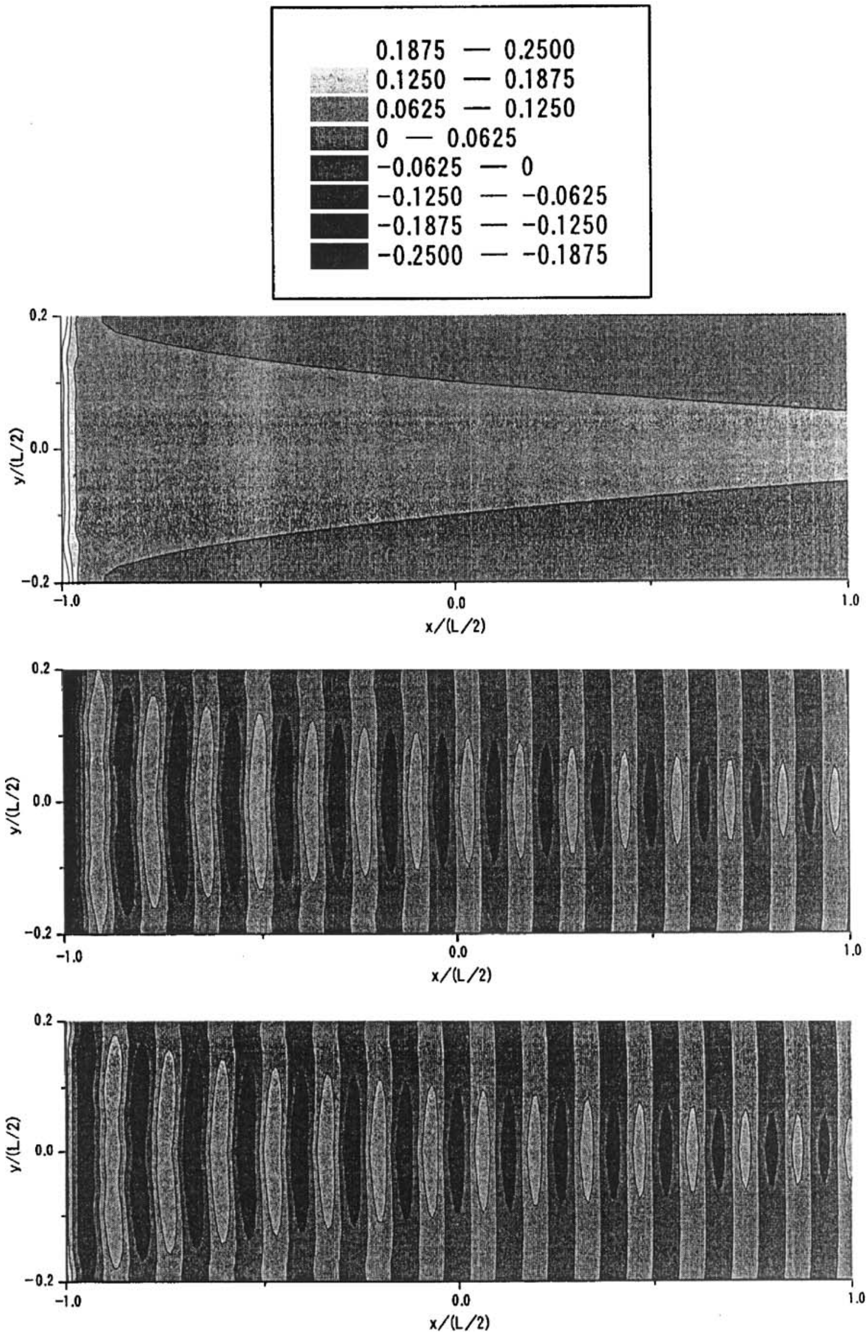

Fig. 3. Amplitude (upper), imaginary part (middle), and real part (lower) of deflection of VLFS in head seas, $KL = 100\pi$, $h/L = 0.01$.

small fluctuation is naturally not predicted. The reason for the large amplitude predicted by the BE method at the rear end is also explained by the same end effect. Presumably if this effect were correctly taken into account in the present approach, the agreement could be perfect. Except for this disagreement, the present method is able to predict the plate deflection at good accuracy, at the center line in particular. At the side-edge the present method is slightly less accurate.

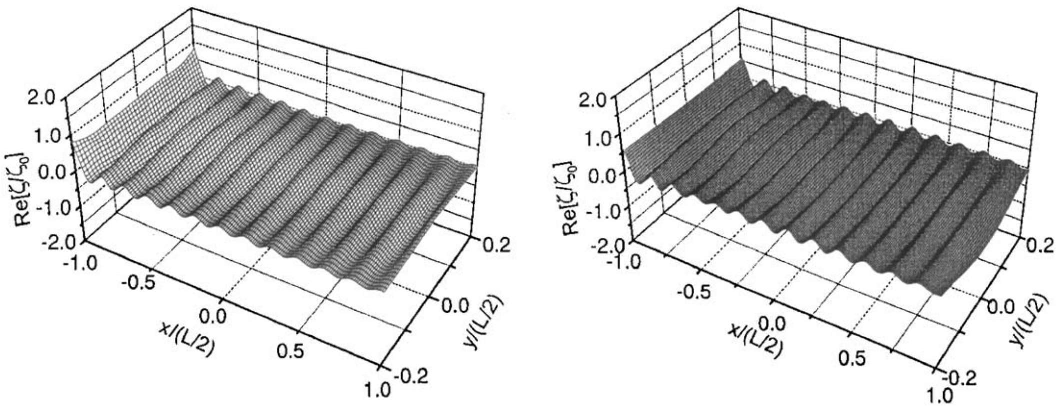

Fig. 4. Definition of VLFS in head seas, $KL = 40\pi/L$, $h/L = 0.01$, left: present method, right: panel method (Kashiwagi, 1998).

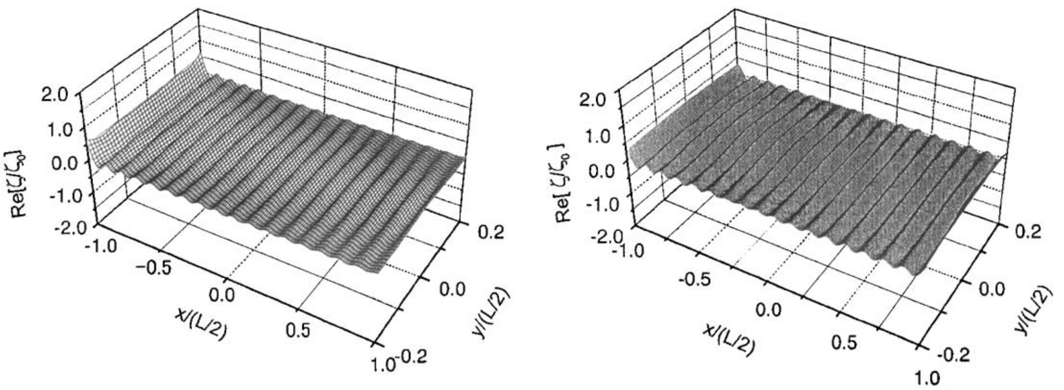

Fig. 5. Definition of VLFS in head seas, $KL = 40\pi/L$, $h/L = 0.002$, upper: present method, lower: panel method (Kashiwagi, 1998).

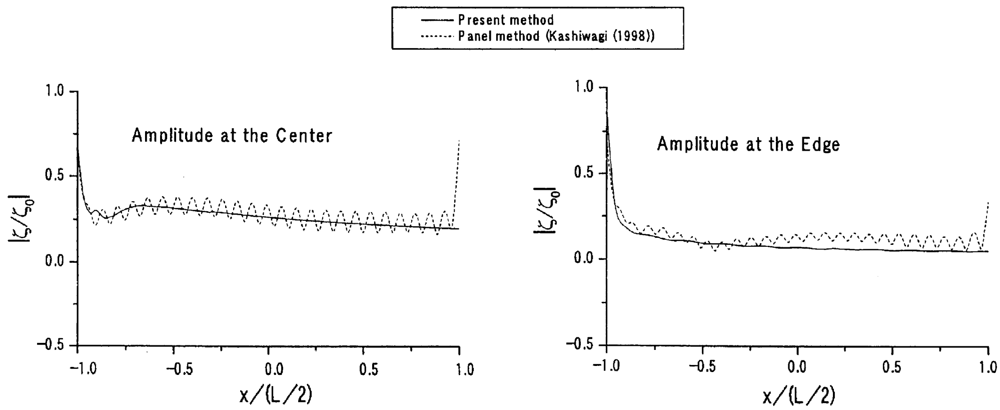

Fig. 6. Deflection of VLFS in head seas, $KL = 40\pi/L$, $h/L = 0.01$.

It is reported in Kashiwagi (1998) that the plate deflection predicted by his BE method agrees well with experimental data. This means that the present method will be also accurate if its results are compared with experimental measurements.

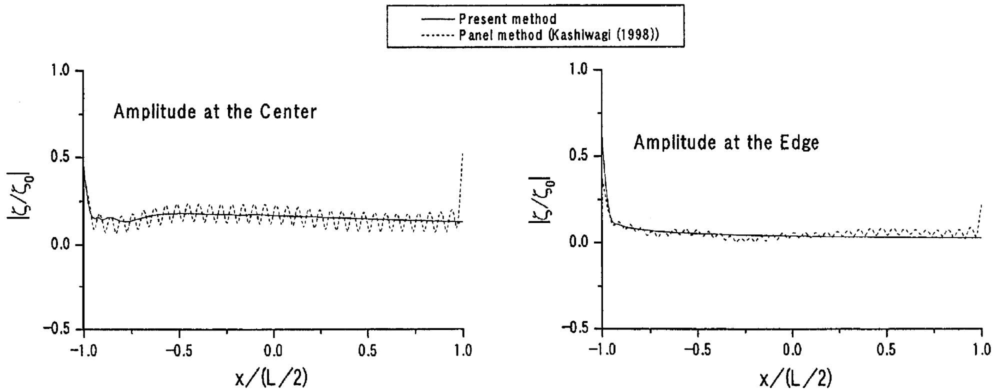

Fig. 7. Deflection of VLFS in head seas, $KL = 40\pi/L$, $h/L = 0.002$.

6. Concluding remarks

A new approach is proposed to analyze the bending vibration of a large floating structure whose form is just like a thin plate. The horizontal size of this plate is huge compared with the wavelength, while the draft is much smaller than the wavelength. The fluid–plate interaction is solved mathematically by considering that the plate bottom surface is located at the water surface. The dynamic deflection of the thin plate is understood as a wave propagating on the plate. In the present approach the main concern is to solve for the wave refraction and reflection at the boundary between the plate bottom surface and the real water surface. We seek the solution by matching the wave in the plate to the wave on the water surface, with the plate edge solution satisfying the conditions of zero bending moment and zero shear force. The plate vibration is obtained in an explicit analytical form, which can be evaluated readily with almost no computational effort. This simple formula will be useful for better understanding of the hydroelasticity in waves of large floating structures of thin plate configuration.

The accuracy of the plate vibration predicted by the present method is confirmed against the results from a rigorous but more computationally involved approach. Of course real structures will be of complicated configuration, and the prediction of their vibration requires a computational approach based on large scale use of computers. Yet the present method will be a bench mark for validating the results of large scale numerical computations.

Appendix A

If the plate shown in Fig. 2 has no edge at $y = 0$, i.e., the plate extends from $y = -\infty$ to $+\infty$, the plate vibration must be uniform with y and Eq. (7) will be simplified to

$$D\frac{d^6\phi}{dx^6} + \rho g\frac{d^2\phi}{dx^2} + \rho g k^2\phi = 0. \tag{A.1}$$

Assume

$$D = \mathcal{O}(k^{-4}), \quad \frac{d}{dx} = \mathcal{O}(k), \tag{A.2}$$

then every term of Eq. (A.1) is of the same order $\mathcal{O}(k^2)$ and the same problem as in Stoker (1958) is recovered. If the plate is stiffer ($D \gg \mathcal{O}(k^{-4})$), then no transverse vibration mode occurs. A more flexible plate will yield a solution that is essentially not different from the one derived from the first of assumptions (A.2).

A solution of Eq. (A.1) is given by

$$\phi = \sum_{j=1}^{6} A_j e^{\lambda_j x}, \tag{A.3}$$

where λ_j are the six roots of

$$D\lambda_j^6 + \rho g\lambda_j^2 + \rho g k^2 = 0. \tag{A.4}$$

These are

$$\lambda_{1,4} = \mp \mathrm{i}k_A, \ \lambda_{2,5} = \mp\lambda, \ \lambda_{3,6} = \mp\lambda^*, \quad -\pi/2 < \arg(\lambda) < \pi/2,$$

where

$$k_A = \sqrt{|u+v|}, \quad \lambda^2 = -\frac{1}{2}\left[(u+v) + \mathrm{i}\frac{\sqrt{3}}{2}(u-v)\right],$$

$$u, v = \left[-\frac{\beta}{2} \pm \left(\frac{\beta^2}{4} + \frac{\alpha^3}{27}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}, \quad \alpha = \frac{\rho g}{D}, \quad \beta = \frac{\rho g k}{D}$$

and the asterisk stands for the complex conjugate. It is straightforward to show $\lambda_j = \mathcal{O}(k)$ and therefore to confirm the legitimacy of the second assumption of Eq. (A.2).

A_j in Eq. (A.3) will be determined as follows: $A_{5,6} = 0$ results from the condition that ϕ is finite at $x = +\infty$ and $A_4 = 0$ because no waves come from $x = +\infty$; the other three $A_{1,2,3}$ are determined by requirement of zero bending moment and zero shear force at $x = 0$:

$$(-\mathrm{i}k_A)^5 A_1 + \lambda_2^5 A_2 + \lambda_3^5 A_3 = 0, \tag{A.5}$$

$$(-\mathrm{i}k_A)^4 A_1 + \lambda_2^4 A_2 + \lambda_3^4 A_3 = 0 \tag{A.6}$$

and the condition of continuity of mass and energy flux at $x = 0$ with the wave solution for $x < 0$:

$$A_1 + A_2 + A_3 = 1 + R, \tag{A.7}$$

$$(-\mathrm{i}k_A)A_1 + \lambda_2 A_2 + \lambda_3 A_3 = -\mathrm{i}k + \mathrm{i}kR, \tag{A.8}$$

where the wave function ϕ at $x < 0$ is represented by

$$\phi = \mathrm{e}^{-\mathrm{i}kx} + R\mathrm{e}^{\mathrm{i}kx}. \tag{A.9}$$

The linear Eqs. (A.5)–(A.8) determine all the unknowns A_1, A_2, A_3 and R.

References

Evans, D.V., Davies, T.V., 1968. Wave–ice interaction. Technical Report 313, Davidson Laboratory, Stevens Institute of Technology, Hoboken, NJ, USA, pp. 1–101.

Faltinsen, O.M., 1990. Sea Lords on Ships and Offshore Structures. Cambridge University Press, Cambridge.

Hermans, A., 1997. The excitation of waves in a very large floating flexible platform by short free-surface water waves. Proceedings of 12th International Workshop on Water Waves and Floating Bodies, Carry-le-Rouet, pp. 107–110.

Kashiwagi, M., 1998. A B-spline Galerkin scheme for calculating hydroelastic response of a very large floating structure in wave. Journal of Marine Science and Technology 3, 37–49.

Mei, C.C., Tuck, E.O., 1980. Forward scattering by long thin bodies. SIAM Journal of Applied Mathematics 39, 178–191.

Meylan, M., Squire, V.A., 1994. The responses of ice floe to ocean waves. Journal of Geographic Research 99, 891–900.

Namba, Y., Ohkusu, M., 1999. Hydroelastic behavior of floating artificial islands in waves. International Journal of Offshore and Polar Engineering 9, 39–47.

Newman, J.N., Maniar, H.D., Lee, C.H., 1996. Analysis of wave effect for very large floating structures. Proceedings of International Workshop on Very Large Floating Structures, Hayama, Japan, pp. 135–142.

Ohkusu, M., Namba, Y., 1996. Hydroelastic behavior of a very large floating platform in waves. In: Proceedings of 11th International Workshop on Water Waves and Floating Bodies, Hamburg, Germany.

Ohkusu, M., Namba, Y., 1998. Hydroelastic behavior of a large floating platform of elongated form in head waves. Proceedings of Second International Conference on Hydroelasticity in Marine Technology, Fukuoka, Japan, pp. 177–183.

Stoker, J.J., 1958. Water Waves. Wiley, New York.

Takagi, K., 2001. Hydroelastic response of a very large floating structure in waves—a simple representation by the parabolic approximation. Private communication.